# A non-covariant fermionic determinant and its connection to Luttinger systems



2005 J. Phys. A: Math. Gen. 38 749

(http://iopscience.iop.org/0305-4470/38/3/011)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **38** (2005) 749–758 doi:10.1088/0305-4470/38/3/011

# A non-covariant fermionic determinant and its connection to Luttinger systems

**Aníbal Iucci**[1,2] **and Carlos Naón**[1,2]

[1] Instituto de Física La Plata, Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de La Plata, CC 67, 1900 La Plata, Argentina
[2] Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina

E-mail: iucci@fisica.unlp.edu.ar and naon@fisica.unlp.edu.ar

Received 10 September 2004, in final form 12 November 2004
Published 23 December 2004
Online at stacks.iop.org/JPhysA/38/749

**Abstract**

We consider a fermionic determinant associated with a non-covariant quantum field theory used to describe a non-relativistic system in $(1+1)$ dimensions. By exploiting the freedom that arises when Lorentz invariance is not mandatory, we determine the heat-kernel regulating operator so as to reproduce the correct dispersion relations of the bosonic excitations. We also derive the Hamiltonian of the functionally bosonized model and the corresponding currents. In this way, we were able to establish the precise heat-kernel regularization that yields complete agreement between the path-integral and operational approaches to the bosonization of the Tomonaga–Luttinger model.

PACS numbers: 11.10.Ef, 71.10.Pm, 05.30.Fk

## 1. Introduction

Fermionic determinants play a central role in modern formulations of quantum field theories (QFTs). In particular, they arise in the path-integral formulation of fermionic models [1]. In the last 20 years it has been especially fruitful in the study of fermionic determinants in $(1+1)$ dimensions. Fujikawa's observation concerning the non-triviality of the Jacobian associated with chiral changes in the fermionic functional integration measure [2], when specialized to the $(1+1)$-dimensional case, led to significant advances in our understanding of paradigmatic 'toy-models' such as two-dimensional quantum electrodynamics ($QED_2$), the Thirring model and their non-Abelian versions [3]. In fact, a functional bosonization technique was developed on the basis of an adequate treatment of the fermionic determinant [4]. The crucial point is that the above-mentioned Jacobian needs a regularization. For gauge theories with Dirac fermions one is naturally led to consider a regularization scheme that preserves gauge invariance. On the other hand, when the vector fields which are present in the theory are just auxiliary fields (usually introduced through a Hubbard–Stratonovich transformation), one can choose a more

  

general regulator [5, 6]. The Thirring model [7] and the chiral Schwinger model [8] are the examples in which regularization ambiguities take place.

The regularization of the Fujikawa Jacobian and its role in the analysis of quantum anomalies have been extensively examined in the literature [9]. In all the cases the models under study are relativistic QFTs, i.e. Lorentz covariant theories. However, in certain relevant situations one is interested in non-covariant field theories. This is the case in the analysis of one-dimensional (1D) electronic systems which can be studied through the g-ology model [10], a theory with four coupling constants $g_1, \ldots, g_4$ associated with different scattering processes (for $g_1 = g_3 = 0$ it reduces to a low-energy model for electrons known as the Tomonaga–Luttinger model (TLM) [11]). In this context, the existence of a functional bosonization, alternative to the usual operator approach, was first suggested by Fogedby [12] and further elaborated by Lee and Chen [13]. The explicit connection between the functional bosonization leading to an effective action describing the dynamics of bosonic collective excitations and the Fujikawa Jacobian was first established in [14]. But even in this case a covariant regularization, borrowed from relativistic field theory, was employed. As a result, the general expressions for the dispersion relations of the bosonic modes, in terms of the couplings $g_2$ and $g_4$ of the TLM, did not agree with those obtained through conventional, operational bosonization. In this work we show that the origin of this disagreement is in the type of regularization chosen to compute the Fujikawa Jacobian. Since the underlying Lorentz invariance is violated, an almost arbitrary number of regularization schemes seems possible. Only one of them leads to the usual result for the TLM.

In section 2 we present the model and express its generating functional in terms of a fermionic determinant. In section 3, in order to clarify the discussion, we start by sketching the main steps of the decoupling approach to bosonization and the results obtained using a standard Lorentz invariant regularization. We then include two subsections 3.1 and 3.2 where we present two different non-covariant regularizations, the point-splitting and the heat-kernel methods, respectively. In this last case we determine the precise form of the regulating heat-kernel operator needed to obtain the right answer for the dispersion relations. We would like to stress that we were not able to identify a physical guiding principle to choose *a priori* between different regularization schemes. However, as far as we know, such a principle is still not clarified even in the operational bosonization of condensed matter theories. Of course, this is an important issue that deserves further investigations. In section 4 we show how to derive, in our functional bosonization framework, the bosonic Hamiltonian and the corresponding bosonized currents. Finally, we briefly discuss the issue of current conservation. In section 5 we gather our results and conclusions.

## 2. The model and the fermionic determinant

We will consider a non-covariant version of the Thirring model defined by the following Euclidean Lagrangian:

$$\mathcal{L} = \bar{\psi}\, \mathrm{i}\not{\partial}\psi - \frac{g^2}{2} V_{(\mu)}\, j_\mu\, j_\mu, \tag{1}$$

where $V_0$ and $V_1$ are the coupling constants, and the derivatives are redefined in order to include the Fermi velocity,

$$\partial_0 = \frac{\partial}{\partial x_0} \tag{2}$$

$$\partial_1 = v_{\mathrm{F}} \frac{\partial}{\partial x_1}. \tag{3}$$

Note that $v_F$ plays the role of the light velocity in QFT, which is usually taken as unit. For $v_F = 1$ and $V_0 = V_1 = 1$, one has the usual Thirring model (the constant $g^2$ is included to facilitate comparison with the Lorentz invariant results). The fermionic current is defined as

$$j_\mu = \bar{\psi}\gamma_\mu\psi, \tag{4}$$

which satisfies the classical conservation law

$$\partial_\mu j_\mu = 0. \tag{5}$$

The generating functional is

$$\mathcal{Z}[S] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left[-\int \mathrm{d}^2x(\mathcal{L} + j_\mu S_\mu)\right]. \tag{6}$$

By means of a Hubbard–Stratonovich transformation, it can be put in the form

$$\mathcal{Z}[S] = \mathcal{N}\int \mathcal{D}A_\mu \det \not{D}[A] \exp\left[-\frac{1}{2g^2}\int \mathrm{d}^2x\,\mathrm{d}^2y\,V^{-1}_{(\mu)} \times (x-y)(gA_\mu - S_\mu)(x)(gA_\mu - S_\mu)(y)\right], \tag{7}$$

where

$$\not{D}[A] = \mathrm{i}\not{\partial} + g\not{A}, \tag{8}$$

and

$$\det \not{D}[A] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left[-\int \mathrm{d}^2x\bar{\psi}\not{D}[A]\psi\right]. \tag{9}$$

## 3. Decoupling approach to bosonization

Having expressed the generating functional in terms of a fermionic determinant, we shall now sketch the decoupling method which is at the root of the functional approach to bosonization pioneered in [3] (see also [4]). In (1 + 1) space–time the vector field $A_\mu$ can be decomposed into transverse and longitudinal parts:

$$A_\mu = -(1/g)(\epsilon_{\mu\nu}\partial_\nu\phi - \partial_\mu\eta), \tag{10}$$

where $\phi$ and $\eta$ are scalar fields. Let us note that if we perform the following transformation in the fermionic fields:

$$\psi = \mathrm{e}^{t[\gamma_5\phi + \mathrm{i}\eta]}\chi \tag{11}$$

$$\bar{\psi} = \mathrm{e}^{t[\gamma_5\phi - \mathrm{i}\eta]}\bar{\chi}, \tag{12}$$

then the fermionic Lagrangian density changes as

$$\bar{\psi}\not{D}[A]\psi = \bar{\chi}\not{D}_t[A]\chi \tag{13}$$

where

$$\not{D}_t[A] = \not{D}[(1-t)A]. \tag{14}$$

As first observed by Fujikawa [2], the Jacobian associated with the above change in the path integration measure is not trivial, but depends on the fields $\phi$ and $\eta$:

$$\det(\mathrm{i}\not{\partial} + g\not{A}) = J[\phi, \eta; t]\det(\mathrm{i}\not{\partial} + g(1-t)\not{A}). \tag{15}$$

Note that for $t = 1$, the fermionic and bosonic degrees of freedom become completely decoupled. It can be shown that

$$J[\phi, \eta; 1] \equiv J = \exp\left[-\int_0^1 \omega(t)\right] \tag{16}$$

with

$$\omega(t) = -\mathrm{tr}\, \not{D}_t[A]^{-1} g \not{A} = -\lim_{y\to x} \mathrm{tr}^D \int \mathrm{d}^2x \not{D}_t[A]^{-1}(x, y) g \not{A}(x), \tag{17}$$

where $\mathrm{tr}^D$ means the trace in the Dirac space. All these formulae form a close analogy with those corresponding to a covariant QFT. Actually the only difference between them is the presence of $v_\mathrm{F}$ instead of the velocity of light, but this has non-trivial implications. The last equation has to be regularized, otherwise divergences appear, as is obvious by taking the $y \to x$ limit. In QFT, any acceptable regularizing method has to be Lorentz invariant. In the present case we do not have that limitation for two reasons: (i) Lorentz invariance is broken from the beginning since we are in a non-relativistic theory; (ii) there is a remaining covariance, i.e. the theory without interactions is invariant with respect to the Lorentz group where the velocity of light has been replaced by $v_\mathrm{F}$, but this is an artificial symmetry and there is no reason to respect it. Moreover, by taking $V_0 \neq V_1$ ($g_2 \neq g_4$) not even this symmetry is present. Before taking advantage of the freedom arising from the absence of covariance, it could be instructive to recall the results previously obtained by choosing a Lorentz invariant regularization [14]. Using a regulator of the form

$$(\not{D}_t[A]\not{D}_t[A]^\dagger + \not{D}_t[A]^\dagger \not{D}_t[A])/2, \tag{18}$$

which was first proposed by Fujikawa in his analysis of covariant and consistent anomalies [15], one gets

$$J_\mathrm{cov} = \exp\left\{-\frac{a}{2\pi v_\mathrm{F}} \int \mathrm{d}^2x[(\partial_1\phi)^2 + (\partial_0\phi)^2]\right\}, \tag{19}$$

where $a$ is a parameter related to possible regularization ambiguities. For $a = 1$, one has a gauge invariant regularization. Although the Thirring model does not possess local gauge invariance, in the present context we are mainly interested in Lorentz invariance and we can then set $a = 1$ without loss of generality. Inserting $J_\mathrm{cov}$ in the generating functional and expressing $A_\mu$ in terms of $\phi$ and $\eta$ according to equation (10), one obtains a bosonized action. In the condensed-matter context these bosonic degrees of freedom are interpreted as fields associated to charge-density oscillations. From this bosonic action derived through a covariance-preserving regularization, one can easily compute the corresponding dispersion relation:

$$p_0^2 + v_\mathrm{cov}^2 p_1^2 = 0 \tag{20}$$

where

$$v_\mathrm{cov}^2 = v_\mathrm{F}^2 \frac{\left(v_\mathrm{F} + \frac{g^2 V_0}{\pi}\right)}{\left(v_\mathrm{F} + \frac{g^2 V_1}{\pi}\right)}. \tag{21}$$

Let us stress that only for $V_1 = 0$ this velocity agrees with the value obtained by using operational bosonization, which reads

$$v^2 = \left(v_\mathrm{F} - \frac{V_1 g^2}{\pi}\right)\left(v_\mathrm{F} + \frac{V_0 g^2}{\pi}\right). \tag{22}$$

We shall now describe two different methods to regularize the Jacobian that do not preserve Lorentz invariance. Both techniques lead to an effective bosonic action containing the right answer for the dispersion relation.

*3.1. Point-splitting method*

As is well known, the point-splitting regularization method breaks Lorentz invariance explicitly. It consists in a prescription for taking the $y \to x$ limit mentioned before by defining

$$\lim_{y\to x} \not{D}_t[A]^{-1}(x, y) = \tfrac{1}{2}(\lim_{\epsilon\to 0^+} + \lim_{\epsilon\to 0^-})\not{D}_t[A]^{-1}(x_0, x_1; x_0, x_1 + \epsilon), \tag{23}$$

i.e. by taking a symmetric limit in the space variable. We need then the Green function of the Dirac operator, which satisfies

$$\not{D}_t[A]_x \not{D}_t[A]^{-1}(x, y) = \delta^2(x - y). \tag{24}$$

As usual, we propose the ansatz

$$\not{D}_t[A]^{-1}(x, y) = \exp((1 - t)[\gamma_5\phi(x) + \mathrm{i}\eta(x)])G_0(x, y)\exp((1 - t)[\gamma_5\phi(y) - \mathrm{i}\eta(y)]), \tag{25}$$

where $G_0$ is the Green function of the free Dirac operator:

$$\mathrm{i}\not{\partial}_x G_0(x, y) = \delta^2(x - y). \tag{26}$$

With this recipe, we find for equation (23) the following result:

$$\lim_{y\to x} \not{D}_t[A]^{-1}(x, y) = -\frac{\mathrm{i}}{2\pi v_\mathrm{F}}(1 - t)\gamma_1\partial_1[\gamma_5\phi(x) - \mathrm{i}\eta(x)] \tag{27}$$

and then, the Jacobian (equations (16) and (17)) is given by

$$J = \exp\left\{-\frac{1}{2\pi v_\mathrm{F}}\int \mathrm{d}^2x[(\partial_1\phi)^2 - (\partial_1\eta)^2 - 2\partial_1\phi\partial_0\eta]\right\}. \tag{28}$$

The vacuum functional can then be written as

$$\mathcal{Z}[S = 0] = \mathcal{N}\int \mathcal{D}\phi\mathcal{D}\eta\, \mathrm{e}^{-S_\mathrm{eff}} \tag{29}$$

where $\mathcal{N}$ is a normalization factor that includes the free fermion (interaction independent) determinant. We have also defined $S_\mathrm{eff}$, which in momentum space takes the form

$$S_\mathrm{eff} = \int \frac{\mathrm{d}^2p}{(2\pi)^2}[\phi(p)A\phi(-p) + \eta(p)B\eta(-p) + 2\phi(p)C\eta(-p)], \tag{30}$$

with

$$A = v_\mathrm{F}^2 p_1^2\left(\frac{1}{2g^2V_0} + \frac{1}{2\pi v_\mathrm{F}}\right) + \frac{p_0^2}{2g^2V_1} \tag{31}$$

$$B = v_\mathrm{F}^2 p_1^2\left(\frac{1}{2g^2V_1} - \frac{1}{2\pi v_\mathrm{F}}\right) + \frac{p_0^2}{2g^2V_0} \tag{32}$$

$$C = p_1 p_0 v_\mathrm{F}\left(\frac{1}{2g^2V_1} - \frac{1}{2g^2V_0} - \frac{1}{2\pi v_\mathrm{F}}\right). \tag{33}$$

The physical content of the model can be extracted from $S_\mathrm{eff}$ which describes the dynamics of the collective modes of the system. When the original fermionic model is related to the Tomonaga–Luttinger model used to study one-dimensional electronic systems [11], these collective excitations correspond to charge-density oscillations (plasmons). Their dispersion relation can be obtained as the zeros of the determinant of the matrix

$$\begin{pmatrix} A & C \\ C & B \end{pmatrix}. \tag{34}$$

The result is

$$p_0^2 + v^2 p_1^2 = 0 \tag{35}$$

where $v$ is the renormalized velocity of the charge-density modes given by equation (22).

*3.2. Heat-kernel method*

Another popular way of dealing with the regularization of fermionic determinants is the heat-kernel method [2, 16]. In this scheme $J$ is regulated by inserting an operator of the form $\mathrm{e}^{-R/M^2}$, $R$ is a positive definite operator and $M$ is a mass-like parameter which is kept fixed in the intermediate computations. The limit $M^2 \to \infty$ is taken at the end. Again, let us emphasize that in standard QFT contexts the regulating operator $R$ can be chosen among all operators compatible with Lorentz invariance (set aside, for the moment, any other possible symmetries), for instance $R = \not{D}_t[A]^2$. Here we do not have that limitation, and our purpose is to find the precise form of $R$ that leads to an effective action containing the desired dispersion relation.

We start by rewriting equation (17) as

$$\omega(t) = \mathrm{tr}\{\not{D}_t[A]^{-1}[(\gamma_5\phi - \mathrm{i}\eta)\not{D}_t[A] + \not{D}_t[A](\gamma_5\phi + \mathrm{i}\eta)]\}. \tag{36}$$

The trace operation is ill defined, and needs to be regularized. We define our regularized $\omega$ as

$$\omega(t)_R = \lim_{M\to\infty} \mathrm{tr}\{\not{D}_t[A]^{-1}[(\gamma_5\phi - \mathrm{i}\eta)\not{D}_t[A] + \not{D}_t[A](\gamma_5\phi + \mathrm{i}\eta)]\,\mathrm{e}^{-R/M^2}\}. \tag{37}$$

The choice of $R$ is always dictated by physical considerations. For instance, if we are considering a gauge theory, we must take into account regularization prescriptions which do not spoil gauge invariance at the quantum level. This is usually achieved by taking $R = \not{D}_t[A]^2$, where $A_\mu$ is the gauge field. Here, the model under study is not a gauge theory and, therefore, we have even more freedom to choose the regulator. We shall employ an operator of the form $R = \not{D}_t[B]^2$, where $B_\mu$ is certain vector field to be determined. We can write $\omega(t)_R$ as $\omega(t)_R = \omega_0(t) + \omega_{\mathrm{nc}}(t)$ where

$$\omega_0(t) = \mathrm{tr}\left(2\gamma_5\phi\,\mathrm{e}^{-R/M^2}\right) \tag{38}$$

$$\omega_{\mathrm{nc}}(t) = \mathrm{tr}\left\{\not{D}_t[A]^{-1}(\gamma_5\phi - \mathrm{i}\eta)\left[\not{D}_t[A], \mathrm{e}^{-R/M^2}\right]\right\}. \tag{39}$$

Here the subscript 0 indicates the term that we would have obtained if we had employed the cyclic property of the trace in equation (36). The subscript nc refers to a 'non-cyclic' term (this issue is discussed in detail [6]). The final expressions for these two terms are

$$\omega_0(t) = -(1-t)\frac{g}{\pi}\int \mathrm{d}^2x\,\phi\epsilon_{\mu\nu}\partial_\mu B_\nu \tag{40}$$

$$\omega_{\mathrm{nc}}(t) = -(1-t)\frac{g}{2\pi}\int \mathrm{d}^2x\,\partial_\mu(B_\nu - A_\nu)(\epsilon_{\nu\mu}\phi + \delta_{\nu\mu}\eta). \tag{41}$$

At this point it is straightforward to check that taking

$$B_0 = A_0 \tag{42}$$

$$B_1 = -A_1, \tag{43}$$

we get the same results obtained in the previous section (equations (28)–(34)). Thus, we have found an explicit form for the regulating operator of a Jacobian associated with a non-covariant fermionic determinant. This form, in turn, leads to the correct dispersion relation for the Tomonaga–Luttinger model. This is our main result. Let us mention that the non-covariant Jacobian given by equation (28) has been employed as an ansatz in previous works on functional bosonization of Luttinger liquids [17]. The derivation of this Jacobian was one of the principal motivations of the present work.

## 4. Bosonized Hamiltonian and currents

Up to this point we have worked in the Lagrangian formulation, but in condensed-matter applications the Hamiltonian framework is frequently preferred. It is then desirable to show how to derive, in the functional bosonization framework discussed in this paper, the usual Hamiltonian for the one-dimensional electronic system, i.e. the bosonic form of the Tomonaga–Luttinger model [11]. The other point we address in this section is the bosonic form of the original fermionic currents (charge-density and electrical current) and the issue of conservation.

Taking into account the expression for $\det \not{D}[A]$ calculated in the preceding sections, and the relation between the $\phi$ and $\eta$ fields and the $A_\mu$ fields (equation (10)), we can express the generating functional (7) in terms of the $A_\mu$ field:

$$\begin{aligned}\mathcal{Z}[S] = \mathcal{N}\int \mathcal{D}A_\mu \exp\left(-\frac{1}{2}\int \mathrm{d}^2x\,\mathrm{d}^2y A_\mu(x) D_{\mu\nu}(x-y) A_\nu(y)\right)\\ \times \exp\left(-\frac{1}{2g^2}\int \mathrm{d}^2x\,\mathrm{d}^2y S_\mu(x) V^{-1}_{(\mu)}(x-y) S_\mu(y)\right)\\ \times \exp\left(-\frac{1}{g}\int \mathrm{d}^2x\,\mathrm{d}^2y A_\mu(x) V^{-1}_{(\mu)}(x-y) S_\mu(y)\right),\end{aligned} \tag{44}$$

where $D_{\mu\nu}$ is given in Fourier space by

$$D_{\mu\nu}(p) = \frac{g^2}{\pi\left(p_0^2 + v_\mathrm{F}^2 p_1^2\right)}\begin{pmatrix} v_\mathrm{F} p_1^2 & p_0 p_1 \\ p_0 p_1 & -v_\mathrm{F} p_1^2\end{pmatrix} + \begin{pmatrix}\frac{1}{V_0} & 0 \\ 0 & \frac{1}{V_1}\end{pmatrix}. \tag{45}$$

We can decouple the $A_\mu$ field from the source $S_\mu$ by the usual procedure of performing a translation in the $A_\mu$ field:

$$A_\mu \to A_\mu + \frac{D^{-1}_{\mu\nu} S_\nu}{g V_{(\nu)}}, \tag{46}$$

obtaining

$$\begin{aligned}\mathcal{Z}[S] = \mathcal{N}\int \mathcal{D}A_\mu \exp\left(-\frac{1}{2}\int \mathrm{d}^2x\,\mathrm{d}^2y A_\mu(x) D_{\mu\nu}(x-y) A_\nu(y)\right)\\ \times \exp\left[\frac{1}{2}\int \mathrm{d}^2x\,\mathrm{d}^2y S_\mu(x)\Delta^{-1}_{\mu\nu}(x-y) S_\nu(y)\right]\end{aligned} \tag{47}$$

where $\Delta^{-1}_{\mu\nu}$ is given in Fourier space by

$$\Delta^{-1}_{\mu\nu}(p) = \frac{1}{\pi\left(p_0^2 + v^2 p_1^2\right)}\begin{pmatrix} -K v p_1^2 & p_0 p_1 \\ p_0 p_1 & \frac{v}{K} p_1^2\end{pmatrix}, \tag{48}$$

and the stiffness constant $K$ is given by

$$K = \sqrt{\frac{v_\mathrm{F} - g^2 V_1/\pi}{v_\mathrm{F} + g^2 V_0/\pi}}. \tag{49}$$

We can multiply and divide by

$$\int \mathcal{D}A_\mu \exp\left(-\frac{1}{2}\int \mathrm{d}^2x\,\mathrm{d}^2y A_\mu(x)\Delta_{\mu\nu}(x-y) A_\nu(y)\right), \tag{50}$$

and perform an additional translation in the $A_\mu$ field

$$A_\mu \to A_\mu + \Delta^{-1}_{\mu\nu} S_\nu, \tag{51}$$

to obtain

$$\mathcal{Z}[S] = \tilde{\mathcal{N}} \int \mathcal{D}A_\mu \exp\left(-\frac{1}{2}\int \mathrm{d}^2x\, \mathrm{d}^2y A_\mu(x)\Delta_{\mu\nu}(x-y)A_\nu(y) + \int \mathrm{d}^2x S_\mu(x)A_\mu(x)\right). \tag{52}$$

Finally, by defining the fields $\varphi$ and $\theta$ in the following way:

$$A_0 = \frac{-1}{\sqrt{\pi}}\partial_x\varphi \tag{53}$$

$$A_1 = \frac{\mathrm{i}}{\sqrt{\pi}}\partial_x\theta, \tag{54}$$

we end up with the following generating functional:

$$\mathcal{Z}[S] = \bar{\mathcal{N}} \int \mathcal{D}\varphi\mathcal{D}\theta \exp\left(-\frac{1}{2}\int \mathrm{d}x\, \mathrm{d}\tau \left[\frac{v}{K}(\partial_x\varphi)^2 + vK(\partial_x\theta)^2 + 2\mathrm{i}\partial_x\theta\partial_\tau\varphi\right]\right)$$
$$\times \exp\left(\int \mathrm{d}x\, \mathrm{d}\tau[-S_0\partial_x\varphi/\sqrt{\pi} + \mathrm{i}S_1\partial_x\theta/\sqrt{\pi}]\right). \tag{55}$$

We then naturally identify the $\varphi$ field with the charge-density mode of the system and $\Pi = \partial_x\theta$ as its canonical conjugate field. Moreover, the first two terms in the quadratic action of the previous expression can be identified with the Hamiltonian of the system:

$$\mathcal{H} = \frac{1}{2}\int \mathrm{d}x \left[\frac{v}{K}(\partial_x\varphi)^2 + vK(\partial_x\theta)^2\right], \tag{56}$$

which exactly coincides with the Hamiltonian obtained using standard operational bosonization [11]. Now, by functional derivation we get the bosonic form of the currents

$$j_0 = \frac{-1}{\sqrt{\pi}}\partial_x\varphi \tag{57}$$

$$j_1 = \frac{\mathrm{i}}{\sqrt{\pi}}\Pi, \tag{58}$$

which, of course, are identical to those obtained in the operator approach. It is important to stress that these currents do not obey the continuity equation. Following [17], one introduces a physical electric current $j$, which is in general different from $j_1$. The charge density is identified with $j_0$ ($j_0 = \rho$). The physical current is determined by demanding that the conservation law be verified:

$$\frac{\partial\rho}{\partial\tau} + \frac{\partial j}{\partial x} = 0. \tag{59}$$

We obtain

$$j = \frac{\mathrm{i}}{\sqrt{\pi}}vK\Pi. \tag{60}$$

Note that only for $V_1 = 0$ ($g_2 = g_4$ in the Tomonaga–Luttinger language) one has $vK = 1$

and $j = j_1$. As explained in [17], this difference between $j$ and $j_1$ is due to the fact that, in general, the density does not commute with the interactions.

## 5. Conclusions

We considered a fermionic determinant associated with a non-covariant field theory. In particular, we studied the determinant which arises when implementing a path-integral approach to bosonization based on the decoupling of the fermionic determinant through appropriate changes of variables in the functional integration measure. The model analysed in this work (a non-covariant version of the Thirring model) has been previously used to describe one-dimensional highly correlated electronic systems which display the so-called Luttinger liquid behaviour.

In the context of the heat-kernel regularization method, by exploiting the freedom originated in the non-covariance, we determined a regulating operator that yields a bosonic action which leads to the general form (in terms of the various coupling constants) for the dispersion relations. These dispersion relations are in full agreement with those that are well known in the operational framework. Previous path-integral computations had used a covariant regularization, borrowed from relativistic field theory, which gives a correct spectrum only for particular values of the coupling constants.

We showed how to derive the bosonized Hamiltonian and currents, which coincide with those obtained through standard operational bosonization. In this way we were able to establish the precise heat-kernel regularization that yields complete agreement between the path-integral and operational approaches to the bosonization of the Tomonaga–Luttinger model.

## Acknowledgments

This work was partially supported by Universidad Nacional de La Plata, Consejo Nacional de Investigaciones Científicas y Técnicas, CONICET and Fundación Antorchas.

## References

[1] Ramond P 1996 *Field Theory, A Modern Primer* (Cambridge: Cambridge University Press)
[2] Fujikawa K 1979 *Phys. Rev. Lett.* **42** 1195
Fujikawa K 1980 *Phys. Rev.* D **21** 2848
Fujikawa K 1980 *Phys. Rev.* D **22** 1499 (erratum)
[3] Roskies R and Schaposnik F 1981 *Phys. Rev.* D **23** 558
Gamboa Saraví R E, Schaposnik F A and Solomin J E 1981 *Nucl. Phys.* B **185** 239
Furuya K, Gamboa Saraví R E and Schaposnik F A 1982 *Nucl. Phys.* B **208** 159
[4] Stone M 1994 *Bosonization* (Singapore: World Scientific)
[5] Rubin M 1986 *J. Phys. A: Math. Gen.* **19** 2105
[6] Cabra D and Schaposnik F 1989 *J. Math. Phys.* **30** 816
[7] Klaiber B 1968 *Lectures in Theoretical Physics, Boulder* ed A Barut (New York: Gordon and Breach)
[8] Jackiw R and Rajaraman R 1985 *Phys. Rev. Lett.* **54** 1219
Jackiw R and Rajaraman R 1985 *Phys. Rev. Lett.* **55** 224
[9] Fujikawa K and Suzuki H 2003 Anomalies, local counterterms and bosonization *Preprint* hep-th/0305008
[10] Solyom J 1979 *Adv. Phys.* **28** 209
[11] Voit J 1995 *Rep. Prog. Phys.* **58** 977
Schulz H J, Cuniberti G and Pieri P 1998 *Preprint* cond-mat/9807366
Rao S and Sen D 2000 *Preprint* cond-mat/0005492
Varma C M, Nussinov Z and van Saarloos W 2001 *Preprint* cond-mat/0103393
[12] Fogedby H C 1976 *J. Phys. C: Solid State Phys.* **9** 3757
[13] Lee D K K and Chen Y 1988 *J. Phys. A: Math. Gen.* **21** 4155

[14] Naón C M, von Reichenbach M C and Trobo M L 1995 *Nucl. Phys.* B **435** 567
[15] Fujikawa K 1985 *Phys. Rev.* D **31** 341
[16] Gamboa Saraví R E, Muschietti M A, Schaposnik F A and Solomin J E 1984 *Ann. Phys. (NY)* **157** 360
[17] Fernández V, Iucci A and Naón C 2002 *Nucl. Phys.* B **636** 514
Fernández V, Iucci A and Naón C 2002 *Eur. Phys. J.* B **30** 53
Naón C M and Salvay M 2003 *Nucl. Phys.* B **663** 591
[18] Giamarchi T 1991 *Phys. Rev.* B **44** 2905